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Turbulent shear layer reattachment downstream of a backward-facing step in confined supersonic axisymmetric flow

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The reattachment of a supersonic jet with a turbulent boundary layer abruptly expanding into an axisymmetric parallel diffuser has been experimentally investigated using a surface-flow technique. Measurements were made in the started condition, where the blowing pressure is sufficiently high to establish an oblique shock system in the diffuser. The proposed reattachment criterion correlates present measurements in terms of the diffuser area ratio, and also those of other workers for unconfined flow in terms of the free stream Mach number after separation. As already reported for unconfined flow, it is found that disturbances downstream of reattachment do not affect the upstream region.

Introduction

Considerable theoretical and experimental work has been reported on evaluation of base pressure in supersonic flow over two-dimensional and axisymmetric backward-facing steps. Almost all the theoretical investigations are based on the Korst–Chapman model (figure 1), where the flow field is divided into four regions. (Figure 1 also shows the associated velocity profiles.) In region I, a supersonic free stream with boundary layer approaches the step and undergoes abrupt expansion to the base pressure p_2. In region II, the stream entrains and mixes at constant pressure with the fluid in the separated region. In the mixing or free shear layer, a dividing streamline can be distinguished, above which the main stream fluid is conserved. During recompression in regions III and IV, the dividing streamline stagnates at the wall on reattachment. High-velocity fluid above this streamline overcomes the pressure rise to escape downstream; the entrained fluid below the same streamline is simultaneously returned to the base cavity. (Here, p denotes static pressure; the subscripts 1, 2 and 3 denote the nozzle exit section or separating edge, the base region (also at diffuser entry) and the region downstream of reattachment, respectively; the subscript r denotes the dividing streamline reattachment; and the subscript a denotes the ambient atmosphere.)

Since the base pressure p_2 is determined mainly by the ability of the free shear layer to negotiate the pressure rise to reattachment, Korst (1956) proposed an 'escape criterion', where the total head of the dividing streamline at reattachment is assumed equal to the static pressure far downstream. Thus, only fluid with sufficient total head to exchange for static pressure can flow out of the recompression region. With restriction to 'similar' velocity profiles in the constant pressure mixing region (which are obtained when the initial boundary-layer thickness δ is zero), Korst (1956) used the 'escape criterion' to predict base pressure in two-dimensional, turbulent, supersonic flow. The agreement between predictions and measurements for extremely thin initial boundary layers was good.

While extending Korst's (1956) analysis to include the effects of an initial turbulent boundary layer Nash (1962) found that the 'escape criterion' was inadequate. He suggested that earlier agreement with measurements was caused by a fortuitous cancellation of errors mainly due to (*a*) neglect of the initial boundary layer, and (*b*) inaccurate assumptions regarding the reattachment pressure of the dividing streamline. To obtain agreement between theory and experiment, it was necessary to assume that the dividing streamline stagnated to the local static pressure p_r at the reattachment point, rather than p_3 (figure 1). Nash (1962) defined a reattachment parameter (a reattachment pressure-rise ratio), N, as follows:

$$N = \frac{p_r - p_2}{p_3 - p_2}, \tag{1}$$

and suggested a value of 0·35 based on available measurements. The introduction of N and the effect of the initial boundary layer on shear-layer development into the analysis considerably improved the agreement. Nash's later unpublished work, however, implies that N varies with Mach number M and with the ratio of momentum thickness of the initial boundary layer θ to the base height H.

McDonald (1964) suggested tracing the turbulent shear-layer development from upstream of the separating edge to downstream of reattachment with assumed values of base pressure. The base pressure is then uniquely determined by satisfying the assumption that the final boundary layer emerging from reattachment is of the flat-plate type, and therefore characterized by its shape parameter. The fixed value of 1·4 proposed by McDonald (1964) for the shape parameter was subsequently shown by Roberts (1964) to be inadequate, since it is a function of Mach number. Based on available experimental evidence, Roberts (1964) proposed a reattachment parameter Q, and correlated it in terms of M_2 as follows:

$$Q = \frac{M_3}{M_r} = f(M_2), \tag{2}$$

for two-dimensional turbulent flow. Assuming oblique-shock recompression, Roberts estimated M_3 for given M_2 and flow-turning angle at reattachment, and then evaluated M_r from (2). The reattachment pressure p_r, obtained from isentropic tables for given M_r, yielded good agreement between theory and experiment for $M_1 \leqslant 2{\cdot}0$.

Page, Kessler & Hill (1967) described a 'free-reattachment' model, where it is assumed that the hypothetical flow angle of the adjacent inviscid stream at reattachment is proportional to the flow angle of the approaching shear layer. Based on available experimental data, these workers correlated the constant of proportionality as a function of the non-dimensional velocity along the dividing streamline before reattachment. This correlation has been used by Przirembel & Page (1968) to evaluate the base pressure of an axisymmetric body in supersonic turbulent flow; the calculated pressures compared favourably with measurements.

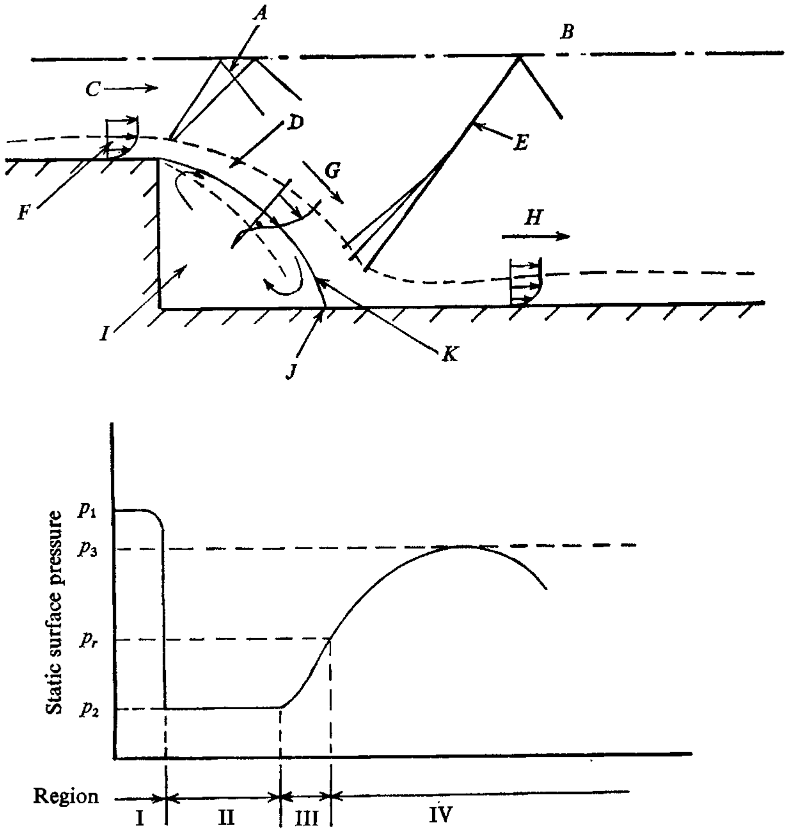

FIGURE 1. Korst–Chapman flow model for confined flow. *A*, expansion fan; *B*, diffuser axis; *C*, M_1; *D*, free shear layer; *E*, oblique shock; *F*, initial boundary layer; *G*, M_2; *H*, M_3; *I*, base region; *J*, reattachment point; *K*, dividing streamline.

Bauer (1964) and Delery (1965) proposed empirical angular laws of reattachment which, in conjunction with Korst's (1956) analysis, were shown to yield base pressures that agreed satisfactorily with measurements in axisymmetric confined flow. However, the procedure to calculate the angular reattachment criterion is necessarily somewhat complicated when the results are required in terms of pressures.

In unconfined flows, to which most of the results so far referred to apply, the conditions downstream of reattachment are uniform. In confined flow this is not so, because of reflected waves, and these may affect the results. The evidence of Sirieix, Mirande & Delery (1966) suggests that artificially introduced disturbances downstream of reattachment have little effect on flow in the base region. This is strongly supported by the experimental investigation of Roshko & Thomke (1966) into the turbulent shear-layer reattachment on the circumference of an axisymmetric body downstream of a backward-facing step in supersonic flow. These workers studied the effect of varying step height (0·25 to 1·68 in.) and free stream Mach number (2·0 to 4·5) on the location of reattachment of the dividing streamline, and wall static pressure at the same point. Their surface pressure distributions suggest a critical condition immediately downstream of reattachment, which renders the flow at reattachment and upstream independent of that further downstream. One object of the present investigation is to confirm that this is also true with the downstream disturbances that are inevitably present in confined flow.

Two methods were used by Roshko & Thomke (1966) to locate the point of reattachment. One was a surface-flow technique using a coating of titanium dioxide in oil to locate the apparently well-defined line of flow reversal. The other technique employed a series of orifice dams, where a small obstruction was cemented just upstream of each orifice along the surface. This combination roughly approximates to a surface Pitot (Preston) tube. Observations of the position of reattachment by the two techniques differed: that determined by the orifice-dam technique lay upstream of the line of flow reversal indicated by the surface-flow technique. The pressure rise to reattachment by the former method was therefore less than that obtained by the latter.

Clearly, more information on the reattachment phenomenon is required, particularly in confined axisymmetric flow where a supersonic jet abruptly expands into an axisymmetric parallel duct, and the flow in the zone of reattachment is turned along the duct wall by an oblique shock. For such flow geometries the experiments of Korst, Chow & Zumwalt (1959), Baker & Martin (1965), Martin & Mukerjee (1968) and Anderson & Williams (1968) show that

$$p_2/P_i = f_1(A_2/A^*, M_1, M_2), \tag{3}$$

where P denotes stagnation pressure, A area, the subscript i denotes the nozzle inlet, and the superscript * denotes the nozzle throat. The observations of Sirieix *et al.* (1966) and Roshko & Thomke (1966) suggest that the reattachment pressure rise, p_r/P_i, can likewise be represented by the following:

$$p_r/P_i = f_2(A_2/A^*, M_1, M_2). \tag{4}$$

To describe the pressure rise to reattachment, a reattachment criterion is defined as

$$R_c = \frac{p_r/P_i - p_2/P_i}{1 - p_2/P_i}. \tag{5}$$

From (3) and (4), the above may be rewritten as

$$R_c = f_3(A_2/A^*, M_1, M_2). \tag{6}$$

At present this relationship can be determined only from the type of experiment described in this paper. It is found that the reattachment criterion, as defined above, yields correlations, one of which holds equally well for both confined and unconfined flows, within the experimental range of Mach number.

Experimental apparatus and procedure

Air at 75 psia and 520 °R approaches an axisymmetric convergent–divergent nozzle through a 3 in. inside diameter pipe of 36 in. unobstructed length followed by a 9 in. length containing honeycomb flow straighteners. From the nozzle the air abruptly expands into an axisymmetric parallel diffuser (figure 2), in which the air stream is compressed under started conditions by an oblique shock system to the ambient pressure at the diffuser exit.

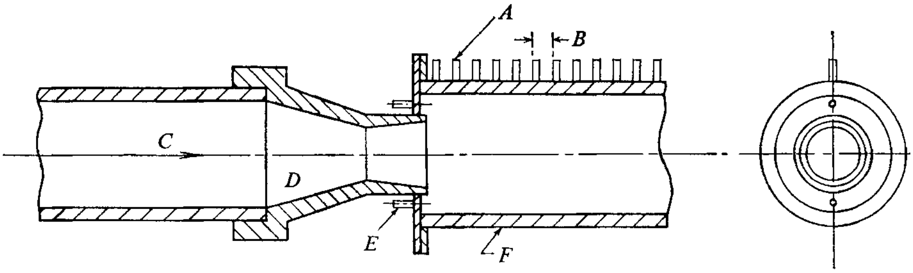

FIGURE 2. Axisymmetric nozzle-diffuser system. *A*, static pressure tap; *B*, 0·25 in. pitch; *C*, air from compressor; *D*, nozzle; *E*, base pressure tap; *F*, Perspex diffuser.

Since, according to (6), R_c is a function of A_2/A^*, M_1 and M_2, the apparatus was designed to examine the effect of these three variables. A bell-shaped nozzle of throat diameter 1·5 in. and actual exit Mach number 1·49 was used, together with three conical nozzles of throat diameter 1·5, 1·4 and 1·27 in., and actual exit Mach numbers of 1·52, 1·7 and 1·95, respectively. When used in conjunction with a series of Perspex diffusers of diameter D_2 of 2·25, 2·625, 2·875 and 3·0 in., and length $L_D \geqslant 6D_2$, area ratios in the range $2{\cdot}18 \leqslant A_2/A^* \leqslant 4{\cdot}6$ could be obtained, with a corresponding variation in base height H between 0·3 and 0·676 in. (the subscript D denoting the diffuser).

Since the dividing streamline stagnates to the local wall static pressure, p_r can be determined from the wall static pressure distribution if the stagnation position is known. To determine this, a thin film of Ragosine grease mixed with yellow dye was evenly applied to the inner surface of the Perspex diffuser over a length of $1{\cdot}5D_2$ from the nozzle exit plane. For a given M_1, sufficient air at $T_{0i} = 520$ °R was admitted to start the diffuser (where T_0 is the stagnation temperature). An axisymmetric flow reversal ring (i.e. the ring of dividing-streamline stagnation) could then be clearly distinguished, whose distance from the nozzle exit plane was measured on an engraved scale.

The corresponding value of p_r was determined by interpolation, where necessary, from the simultaneously recorded wall-static pressure distribution. Further increase in P_i/p_a yielded no measurable displacement of the flow reversal ring, and p_r/P_i remained unchanged. This procedure was repeated for varying M_1 and A_2/A^* in the ranges of $1{\cdot}49 \leqslant M_1 \leqslant 1{\cdot}95$ and $2{\cdot}18 \leqslant A_2/A^* \leqslant 4{\cdot}6$. Limitations in the air supply available precluded investigations at higher M_1 and A_2/A^*.

The flow régime at separation was acertained by measurement of the velocity profile across the nozzle-plane exit. The accuracy of pressure and reattachment location measurements was estimated to be within ± 2 %.

Results and discussion

The measured nozzle-exit velocity profiles for $1{\cdot}52 \leqslant M_1 \leqslant 1{\cdot}95$ illustrated in figure 3 accord sufficiently well with the conventional $\frac{1}{7}$th power law to justify the presumption of a fully turbulent separating boundary layer. Displacement and momentum thicknesses calculated from these profiles also agreed well with those derived from the procedure of Tucker (1951).

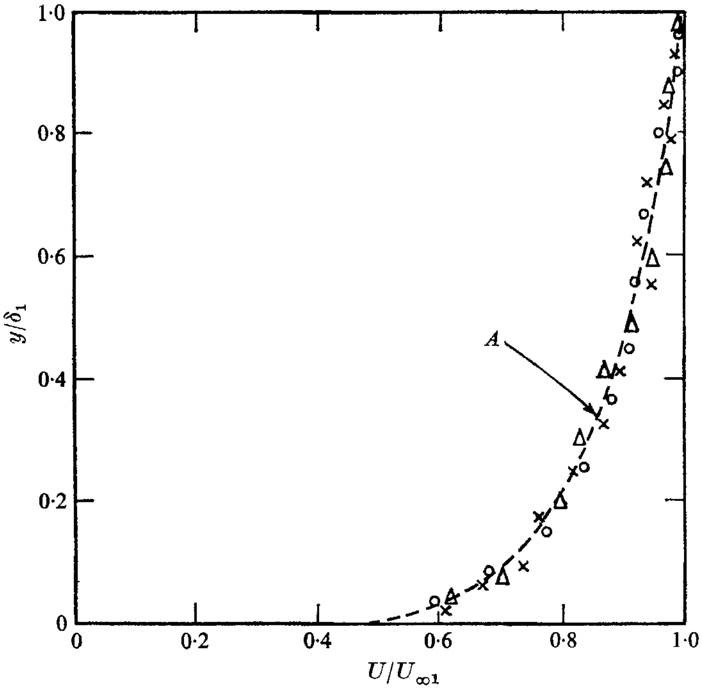

FIGURE 3. Velocity distributions in initial boundary layer at nozzle exit. A is $U/U_\infty = (y/\delta)^{\frac{1}{7}}$. M_1: ○, 1·52; △, 1·7; ×, 1·95. δ_1 (in.): ○, 0·023; △, 0·024; ×, 0·015.

The wall static-pressure distributions, non-dimensionalized by reference to the fixed diffuser discharge pressure p_a, are plotted against x/H in figures 4 and 5 for base heights of 0·3 and 0·485 in., respectively (where x is the distance along the diffuser wall from the nozzle exit plane). The flagged symbols indicate reattachment and stagnation of the dividing streamline. An interesting feature is the complete superimposition of the distributions for different M_1 and δ_1/H (or θ_1/H) in the region of steepest pressure rise. This has also been noted by Roshko &

Thomke (1966), and Hastings (1963) in two-dimensional base flow even when $\delta_1 \gg H$ (base pressures, however, differed considerably). Roshko & Thomke (1966) attribute the phenomenon to the total dependence of the growth of the inner shear layer from the separating edge on the base height. Thus, the dead air or inner part of the flow is mainly governed by developments along the dividing streamline.

By contrast, the effect of M_1 elsewhere in figures 4 and 5 is considerable, the wall static pressure decreasing with increasing M_1. Downstream of reattachment the curves of constant M_1 break away to achieve different maxima for roughly the same value of x, i.e. about 1 in. The subsequent fall in pressure may result from interaction of the expansion fan radiating from the opposite separating edge with the rehabilitated boundary layer downstream of reattachment. The breakaway effect was attributed by Roshko & Thomke (1966) to the influence of lip shocks, which normally recompress the overexpanded flow at the separating edge to the base pressure. Though the lip shocks have been observed by Martin & Baker (1963) in the entry region of the two-dimensional parallel diffuser at $M_1 = 1{\cdot}4$, it appears more likely that the breakaway after reattachment is due

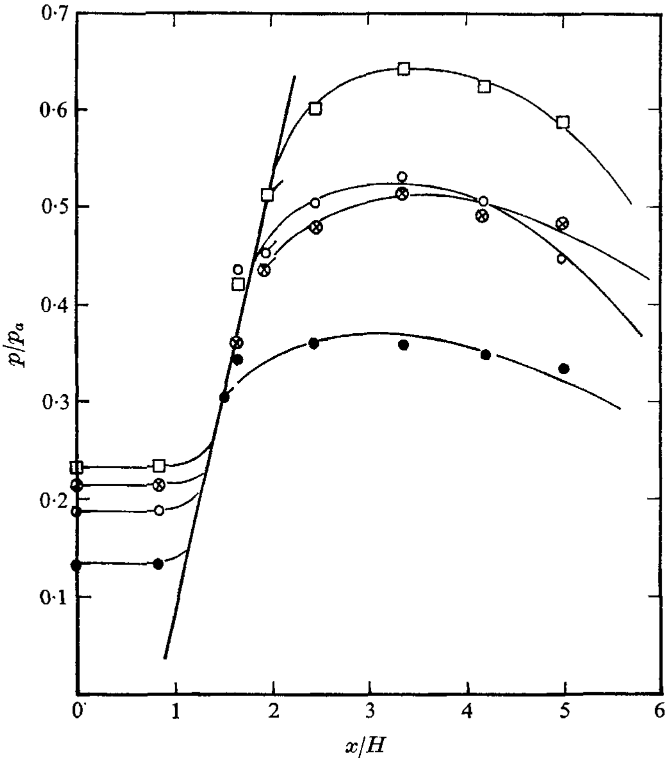

FIGURE 4. Wall pressure distribution at diffuser entry. (Flagged symbols indicate reattachment data.) $H = 0{\cdot}3$ in., $0{\cdot}004 \leqslant \theta_1/H \leqslant 0{\cdot}01$. □, $M_1 = 1{\cdot}49$; ⊗, $M_1 = 1{\cdot}52$; ○, $M_1 = 1{\cdot}7$; ●, $M_1 = 1{\cdot}95$.

to the diminishing strength of the oblique shock since, along its length, the flow turning angle of the approaching streamlines gradually diminishes.

The superimposition of wall static pressure distributions, for a given H, up to dividing-streamline reattachment and the breakaway almost immediately after, suggest that the flow downstream of reattachment has virtually no effect

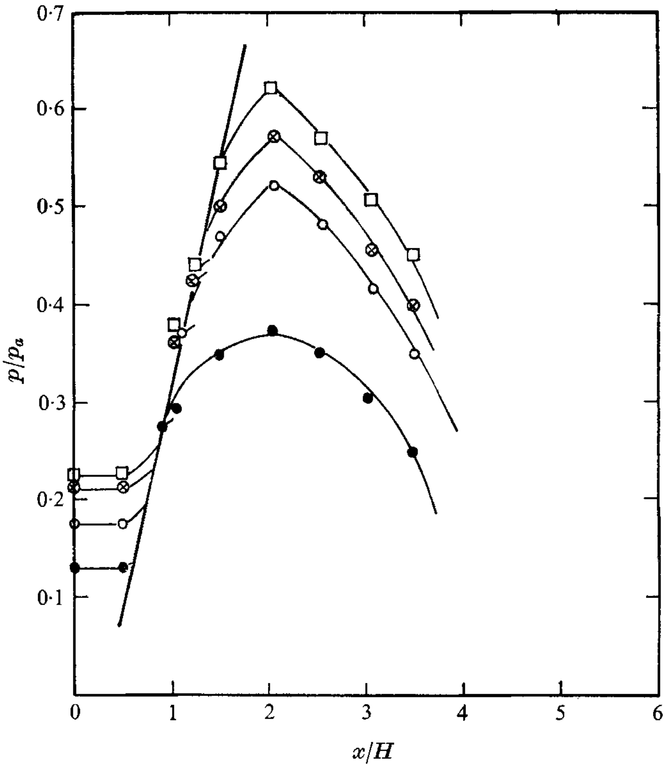

FIGURE 5. Wall pressure distribution at diffuser entry. (Flagged symbols indicate reattachment data.) H = 0·485 in., $0{\cdot}0023 \leqslant \theta_1/H \leqslant 0{\cdot}006$. □, $M_1 = 1{\cdot}49$; ⊗, $M_1 = 1{\cdot}52$; ○, $M_1 = 1{\cdot}7$; ●, $M_1 = 1{\cdot}95$.

on the flow upstream. This implies that at $P_i/p_a > (P_i/p_a)_{\text{start}}$ a critical condition in the reattachment region severs the communication between the base region upstream of reattachment from the region downstream of reattachment (the subscript 'start' denoting the diffuser starting condition). Martin & Mukerjee (1968) and Mukerjee (1968), who used the present reattachment data for base-pressure analysis, find that the critical condition, also observed by Bogdonoff *et al.* (1953), Carrière & Sirieix (1964), Sirieix *et al.* (1966) and Roshko & Thomke (1966), corresponds to the attainment of nearly unit Mach number along the dividing streamline. The stagnation of the dividing streamline will therefore involve a shock that will isolate the base region from disturbances downstream of the reattachment region.

Figure 6 shows the wall static pressure distribution in the steep pressure-rise region near the diffuser entry plotted as p/P_i against x/H for varying H, θ_1/H

and M_1. The best straight line through the region obtained by least squares analysis is represented by the following:

$$p/P_i = 0{\cdot}104(x/H) - 0{\cdot}049. \tag{7}$$

If p is put equal to p_2, the corresponding value of x for given H implies that the length of the constant base-pressure region diminishes with base pressure ratio p_2/P_i, as is also suggested by figures 4 and 5.

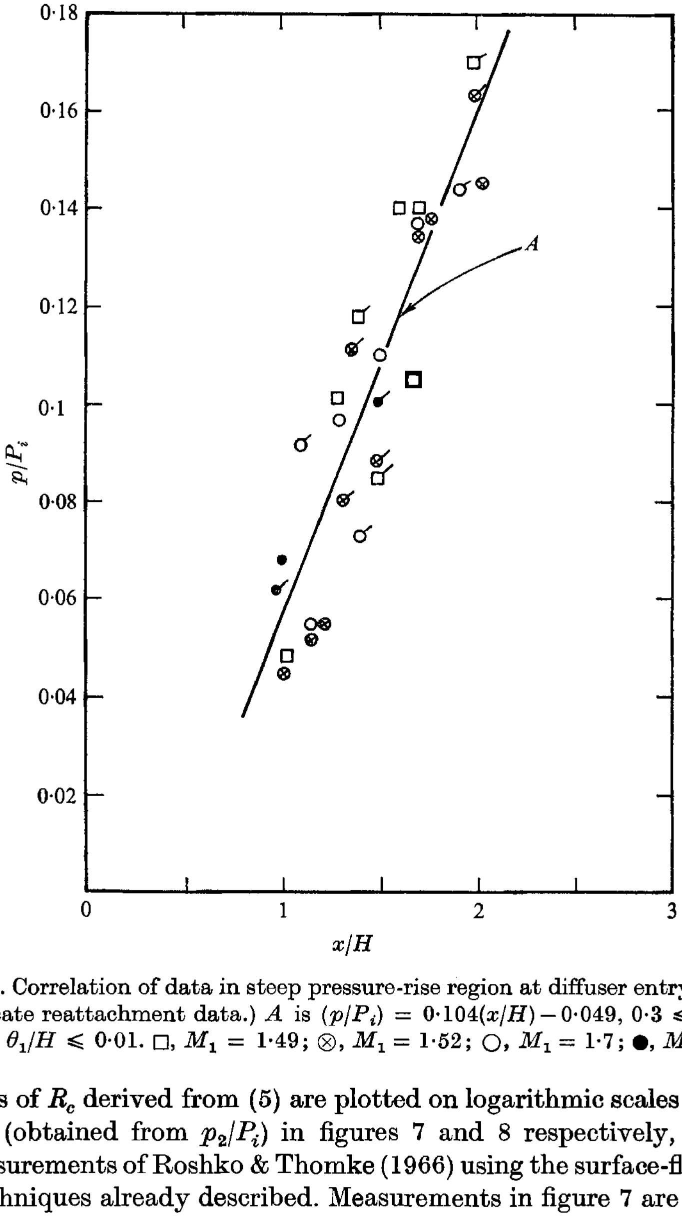

Figure 6. Correlation of data in steep pressure-rise region at diffuser entry. (Flagged symbols indicate reattachment data.) A is $(p/P_i) = 0{\cdot}104(x/H) - 0{\cdot}049$, $0{\cdot}3 \leqslant H \leqslant 0{\cdot}676$ in., $0{\cdot}0023 \leqslant \theta_1/H \leqslant 0{\cdot}01$. □, $M_1 = 1{\cdot}49$; ⊗, $M_1 = 1{\cdot}52$; ○, $M_1 = 1{\cdot}7$; ●, $M_1 = 1{\cdot}95$.

Values of R_c derived from (5) are plotted on logarithmic scales against A_2/A^* and M_2 (obtained from p_2/P_i) in figures 7 and 8 respectively, together with the measurements of Roshko & Thomke (1966) using the surface-flow and orifice-dam techniques already described. Measurements in figure 7 are correlated by

$$R_c = \frac{0{\cdot}314}{(A_2/A^*)^{1{\cdot}475}}, \tag{8}$$

for $2{\cdot}18 \leqslant A_2/A^* \leqslant 4{\cdot}4$. In figure 8, the surface-flow measurements of Roshko & Thomke (1966) and the present measurements are in close accord and both are correlated by

$$R_c = \frac{6{\cdot}0}{M_2^{4{\cdot}74}}, \tag{9}$$

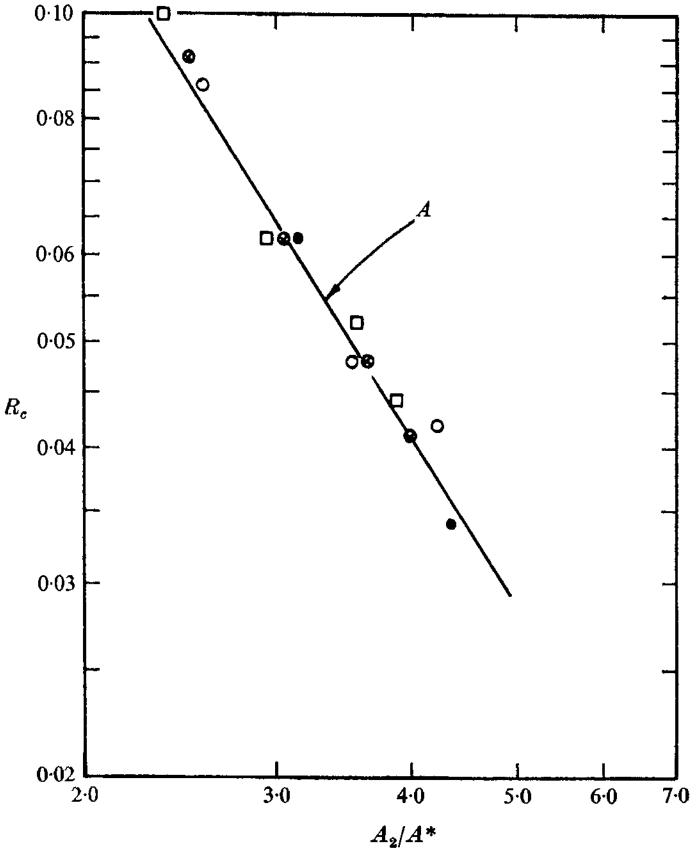

FIGURE 7. Variation of R_c with A_2/A^*. A is $R_c = 0{\cdot}314/(A_2/A^*)^{1{\cdot}475}$
□, $M_1 = 1{\cdot}49$; ⊗, $M_1 = 1{\cdot}52$; ○, $M_1 = 1{\cdot}7$; ●, $M_1 = 1{\cdot}95$.

for $2{\cdot}3 \leqslant M_2 \leqslant 5{\cdot}7$. These measurements differ from the few orifice-dam measurements of Roshko & Thomke (1966), which are best correlated by

$$R_c = \frac{0{\cdot}912}{M_2^{3{\cdot}5}}, \tag{10}$$

for $2{\cdot}3 \leqslant M_2 \leqslant 5{\cdot}7$. For convenience, the above measurements are presented numerically in table 1.

If the orifice-dam measurements are disregarded, the merit of R_c over other reattachment parameters and the applicability of (9), both to confined and unconfined flows over the Mach number range specified, are of considerable importance. The implication is that, as in unconfined flow where the expansion fan from the separating edge does not affect the dividing-streamline reattachment (either directly or after reflexion), for reasons as yet unknown reattachment in confined flow is unaffected by expansion waves generated on the opposite side of the separating edge, though their influence is significant further downstream. This confirms the validity of the assumption of Martin & Mukerjee

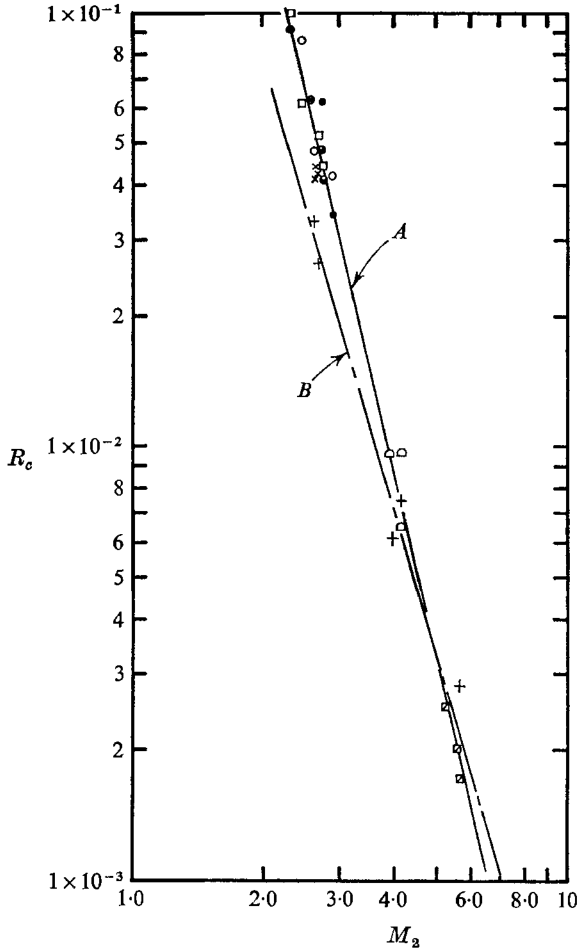

FIGURE 8. Variation of R_c with M_2. A is $R_c = 6\cdot0/M_2^{4\cdot74}$ (surface flow data), B is $R_c = 0\cdot912/M_2^{3\cdot5}$ (orifice-dam data). □, $M_1 = 1\cdot49$; ⊗, $M_1 = 1\cdot52$; ○, $M_1 = 1\cdot7$; ●, $M_1 = 1\cdot95$ (present data); ×, $M_1 = 2\cdot09$; ⌓, $M_1 = 3\cdot02$; ⧄, $M_1 = 3\cdot90$, Roshko & Thomke (surface flow); +, Roshko & Thomke (orifice dam).

(1968) that the Korst–Chapman model, originally developed for unconfined base flow, also holds good for flow at the diffuser entry. There is also further confirmation that the flow upstream of reattachment is unaffected by downstream disturbances even in confined flow.

Conclusions

This experimental investigation of the reattachment phenomena of a supersonic, axisymmetric, turbulent jet, abruptly expanding into a parallel, axisymmetric diffuser, leads to the following conclusions: (i) The base pressure decreases with increasing M_1 for a given H. (ii) The wall static pressure is constant in the base region, but rises steeply up to the stagnation point of the dividing streamline in the free shear layer; thereafter the pressure distribution passes through a maximum determined by M_1. (iii) As suggested by other workers, the wall static pressure expressed as p/P_i in the region of steep pressure rise is independent of H, θ_1/H, M_1 and downstream perturbations. (iv) The proposed reattachment criterion correlates present reattachment measurements in terms of A_2/A^*; it also correlates measurements of other workers, for unconfined flow, in terms of M_2.

M_1	Re' (10^6/in.)	H in.	θ_1/H 10^{-2}	p_2/P_i	M_2	R_c	$(x/H)_{sf}$	$(x/H)_{od}$
1·49†	1·23	0·30	1·03	0·077	2·325	0·1	2·02	—
	1·58	0·48	0·60	0·059	2·495	0·062	1·30	—
	1·88	0·61	0·47	0·044	2·68	0·052	1·56	—
	2·04	0·68	0·42	0·04	2·74	0·044	1·50	—
1·515†	1·22	0·30	0·65	0·077	2·32	0·091	2·0	—
	1·575	0·48	0·38	0·056	2·525	0·062	1·22	—
	1·88	0·61	0·30	0·042	2·715	0·048	1·52	—
	2·03	0·68	0·27	0·039	2·76	0·041	1·3	—
1·7†	1·21	0·30	0·64	0·062	2·46	0·086	1·98	—
	1·58	0·48	0·38	0·044	2·68	0·048	1·03	—
	1·89	0·61	0·293	0·033	2·87	0·042	1·42	—
1·95†	1·1	0·30	0·39	0·039	2·76	0·062	1·50	—
	1·45	0·48	0·23	0·03	2·935	0·034	0·90	—
2·09*	0·0095	0·25	3·89	0·041	2·72	0·044	3·8	—
	0·0095	1·02	0·95	0·043	2·69	0·042	3·34	3·0
	0·0095	1·675	0·58	0·046	2·65	0·041	3·25	3·45
3·02*	0·0136	0·25	4·02	0·0056	4·12	0·0065	2·32	—
	0·0136	1·02	0·99	0·0052	4·17	0·0096	2·65	2·5
	0·0136	1·675	0·06	0·0067	3·99	0·0095	3·1	2·7
3·9*	0·013	0·25	3·59	0·0019	5·23	0·0025	2·3	—
	0·013	1·02	0·88	0·001	5·57	0·002	2·0	—
	0·013	1·675	0·54	0·0009	5·65	0·0017	—	2·4

† Present data. * Roshko & Thomke.

TABLE 1. Summary of experimental data. The subscript 'od' denotes the orifice dam, and the subscript 'sf' denotes the surface flow

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